


# Stark broadening parameters for $6s\ 4F - 6p\ 4(D, F, G)^0$ supermultiplet of singly ionized Hafnium

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**Abstract.** Stark broadening parameters - line widths and shifts - are of interest in stellar spectroscopy as well as in laboratory and technological plasmas. Stark widths and shifts have been calculated for 28 Hf II transitions using plasma electron density of  $10^{17}\text{cm}^{-3}$  and temperature from 5000K to 80000K as parameters. The calculations were performed according to the simplified modified semiempirical (SMSE) method.

**Key words:** Stark broadening – line profiles – atomic data

## 1. Introduction

Insight into impact broadening parameters is of great importance for stellar spectroscopy and laboratory and technological plasma. In this context, stellar abundances must be both precise and accurate in order to obtain reliable information about the stellar atmospheres chemical composition. For example, in metal-poor stars, the strongest and often the only discernible lines of heavy elements are found in ultraviolet (UV) range, from 1000 to 4000 Å and visible blue from 4400 to 4900 Å, Roederer et al. (2012). Absorption lines of interest in these parts of the spectrum are likely to be blended with lines from other sources. To obtain accurate abundances, valid atomic data are necessary, and the electronic level populations in the line-forming layers of the atmosphere must be reliably modeled.

The first determination of the amount of solar hafnium was performed by Russell (1929). The value of  $A(\text{Hf})=0.90$  was surprisingly accurate for the time. In Andersen et al. (1976), the abundance of solar hafnium was redetermined, and reliable transition probabilities for Hf II were obtained. Photospheric spectra were obtained using the McMath Solar Telescope at Kitt Peak National Observatory, Arizona, USA. These analysis was carried out by means of the

method of spectrum synthesis. Lifetime measurements of nine Hf II levels have been performed by the beam-foil technique. These dominant spectral lines of Hf II in the solar spectrum and their upper levels were selected for this investigation. All Hf II lines investigated are situated in regions where numerous absorption lines occur.

The first detection of Hf II in a metal-poor halo star was reported in [Snedden et al. \(1996\)](#) in the spectrum of CS 22892-052 (two Hf II lines at 3719.28 Å and 3793.38 Å), with a mean abundance of  $\log \epsilon = -0.90 \pm 0.10$ . They compared their results with calculated contributions of r- and s-processes in the solar abundances, scaling the solar pattern to best fit the abundances of the  $56 \leq Z \leq 76$  elements. Elemental abundance in CS 22892-052 was reanalyzed by [Snedden et al. \(2003\)](#) who obtained the new value of  $\log \epsilon = -0.98 \pm 0.10$ .

In [Den Hartog et al. \(2021\)](#), authors recorded blue and UV spectra of several metal-poor stars and noticed many lines of singly ionized hafnium. Their report noted new measurements of the branching fraction for 199 UV and optical transitions of Hf II. These transitions range in wavelength (wavenumber) from 2068 to 6584 Å. With these these new transition probabilities it was derived and improved Hf abundances in two metal-poor stars. These lines show potential to be useful abundance indicators, for the stars namely HD 196944 enhanced in s-process elements and HD 222925 enhanced in r-process elements.

Hafnium is very important for nucleosynthesis of heavy elements. Improved laboratory data, especially atomic transition probabilities, are essential for using Hf as a reference element. New results were reported in [Lawler et al. \(2007\)](#), indicating Hf as a suitable stable reference element for nucleocosmochronometry, where it can serve for improved stellar age determination.

In [Lawler et al. \(2007\)](#), authors also disclosed radiative lifetimes of 8 Hf I and 18 Hf II levels, measured with the laser induced fluorescence technique. Branching fractions for transitions from the Hf II levels have been measured from the Fourier transform spectra. Combining the new lifetimes with the Branching fractions, 195 absolute oscillator strengths have been derived. In addition to the Branching fractions, accurate wavelengths for the 195 Hf II lines were measured. These data were applied in the determination of hafnium abundance in the chemically peculiar stars  $\chi$  Lupi (HgMn) and HR 3383 (hot-Am), and were discussed in terms of possible revisions of the hafnium abundance for the Sun and the galactic halo stars CS 22892-052 and CS 31082-001 (see references [Hill et al. \(2002\)](#); [Yushchenko et al. \(2005\)](#); [Ivarsson et al. \(2003\)](#) for details).

## 2. Method

We used the simplified modified semiempirical method [Dimitrijević & Konjević \(1987\)](#), designed for Stark broadening of isolated spectral lines of singly and multiply charged ions in plasma. A more accurate semiclassical perturbation method [Sahal-Bréchet \(1969a,b\)](#); [Sahal-Bréchet et al. \(2014\)](#) is not applicable

in an adequate way due to insufficient atomic data. Precise information on the closest perturber levels for both initial and final states of the transition is crucial for impact broadening parameters calculation.

Accordingly, full width at half maximum intensity reads:

$$w_{smse} = K_{fw} \frac{\lambda^2 N}{\sqrt{T}} \left(0.9 - \frac{1.1}{Z}\right) \sum_{k=i,f} \left(\frac{3n_{l_k}^*}{2Z}\right)^2 (n_{l_k}^{*2} - l_k^2 - l_k - 1) \quad (1)$$

where  $\lambda$ [m] is the wavelength,  $N$ [m<sup>-3</sup>] denotes the perturber density,  $T$ [K] the temperature,  $K_{fw} = 2.21577 \cdot 10^{-20} \text{ m}^2 \text{K}^{1/2}$  is a constant and  $w_{smse}$ [m] is the full width at half maximum intensity. Initial atomic energy level is denoted by  $i$  and the final with  $f$  ( $k = i, f$ ).  $Z$  denotes ion residual charges:  $Z = 1$  for neutral,  $Z = 2$  for singly ionized,  $Z = 3$  for doubly ionized, etc. Effective principal quantum number is labeled  $n_{l_k}^*$ , where  $l_k$  ( $k = i, f$ ) represents orbital angular momentum quantum number.

Formula for the Stark shift calculation depends on whether the transitions with  $\Delta n = 0$  (where  $n$  denotes the main principal number) when all transition for  $l+1$  and  $l-1$  exist or not, as for example for s energy levels, where transition with  $l-1$  is missing. Its general form reads:

$$d_{smse} \approx K_{sh} \frac{\lambda^2 N}{\sqrt{T}} \left(0.9 - \frac{1.1}{Z}\right) \frac{9}{4Z^2} S \quad (2)$$

where  $K_{sh} = 1.1076 \cdot 10^{-20} \text{ m}^2 \text{K}^{1/2}$  is a constant and the polynomial  $S$  is calculated either like:

$$S_1 = \sum_{k=i,f} \frac{n_{l_k}^{*2} \epsilon_k}{2l_k + 1} (n_{l_k}^{*2} - 3l_k^2 - 3l_k - 1) \quad (3)$$

if all transition with  $l+1$  and  $l-1$  exist, or according to:

$$S_2 = \sum_{k=i,f} \frac{n_{l_k}^{*2} \epsilon_k}{2l_k + 1} [(l_k + 1)[n_{l_k}^{*2} - (l_k + 1)^2] - l_k(n_{l_k}^{*2} - l_k^2)] \quad (4)$$

in general;  $\epsilon_k = +1$  for  $k=i$  and  $\epsilon_k = -1$  for  $k=f$ .

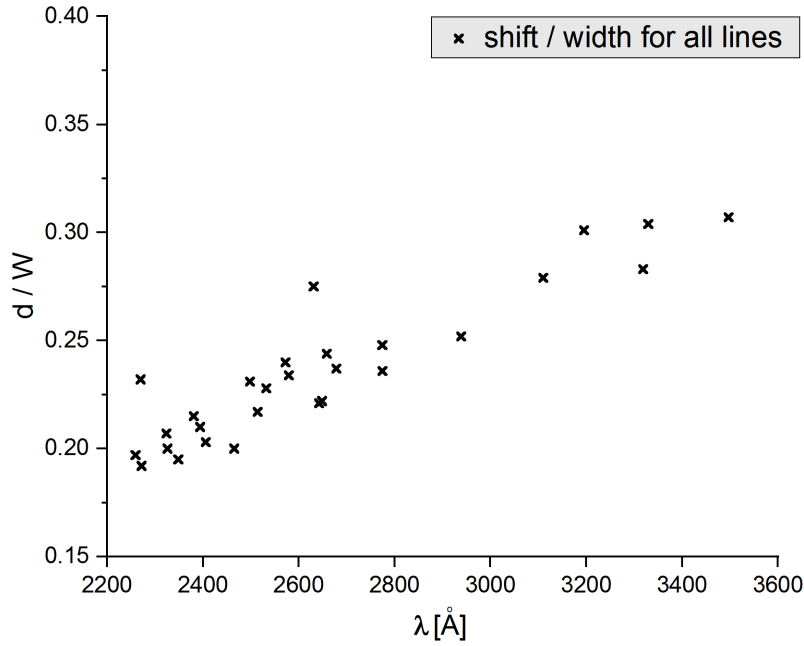
We used the following expression to calculate the averaged energies:

$$E = \frac{\sum_J (2J + 1) E_J}{\sum_J (2J + 1)} \quad (5)$$

where  $E$  represents the averaged energy in units cm<sup>-1</sup>,  $E_J$  energy level and  $J$  the total angular momentum of a particular level.

### 3. Results and discussion

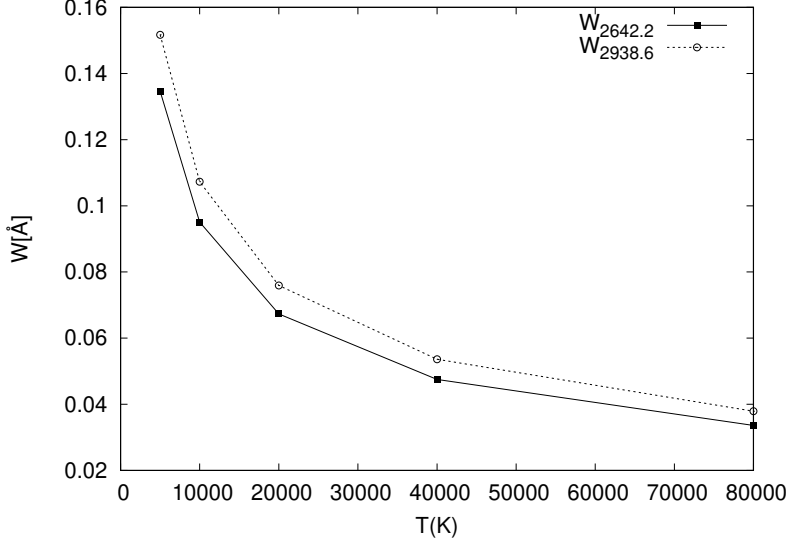
In this contribution, the research was continued on the singly ionized element Hafnium, which expands the list of the heavier elements that we tackled in the previous studies: iridium [Simić et al. \(2021\)](#), rhodium [Simić & Sakan \(2021\)](#) and rhenium [Simić et al. \(2023\)](#). The Stark broadening parameters were obtained - the widths and shifts of 28 Hf II spectral lines, according to the simplified modified semi-empirical method.



**Figure 1.** Ratio of calculated Stark shifts and widths for all spectral lines of singly ionized hafnium.

All calculations were performed using the electron density of  $10^{17}\text{cm}^{-3}$  and temperatures from 5000 to 80000 K. Energy levels were taken from [Moore \(1971\)](#). Our results for the Hf II spectral lines are presented in Table 1. The first column presents transitions with calculated wavelengths, which may differ from the experimental ones. The second one contains the predefined temperatures, and the next two pairs of columns give the corresponding data for Stark width and shift in Å, as well as in angular frequency units. The latter are calculated according to the formulas:

$$W[\text{s}^{-1}] = \frac{2\pi c}{\lambda^2} W \quad (6)$$



**Figure 2.** Dependence of width on temperature for 2642.2 Å and 2938.6 Å spectral lines of singly ionized hafnium of multiplet  $6s\ ^4F - 6p\ ^4G^o$ .

$$d[\text{s}^{-1}] = \frac{2\pi c}{\lambda^2} d \quad (7)$$

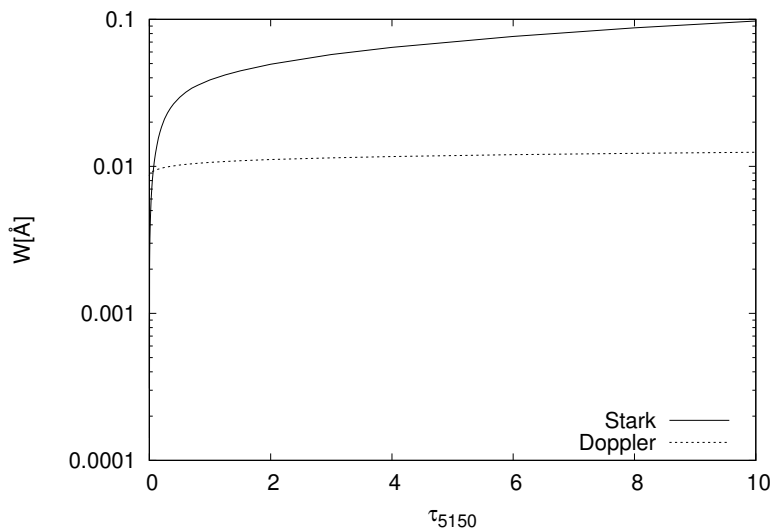
where  $c$  is the light speed in units m/s. The last column gives  $3kT/2\Delta E$ , where  $\Delta E$  is the energy difference between the nearest perturbing level and the closest of the starting and ending levels. The  $3kT/2\Delta E$  term must be less than or equal to two for the method to be valid. To determine the width or shift for an arbitrary experimental wavelength  $\lambda_{exp}$ , the following expressions should be used:

$$W_{exp} = \left(\frac{\lambda_{exp}}{\lambda}\right)^2 W \quad (8)$$

$$d_{exp} = \left(\frac{\lambda_{exp}}{\lambda}\right)^2 d \quad (9)$$

where  $W_{exp}$  and  $d_{exp}$  are the width and shift for an experimental  $\lambda_{exp}$ , and  $\lambda$  is a theoretical wavelength, with corresponding  $W$  and  $d$  (the width and shift) in Table 1.

In our calculations, we considered the supermultiplet of singly ionized hafnium  $5d^2(a^3F)6s\ ^4F - 5d^2(a^3F)6p\ ^4(D, F, G)^o$ , which consists of the following multiplets with the identical parent term  $5d^2(a^3F)$  for both the initial and final level. The first one is  $6s\ ^4F - 6p\ ^4D^o$  with 9 spectral lines, the second one is  $6s\ ^4F - 6p\ ^4F^o$  with 10 spectral lines and the third one is  $6s\ ^4F - 6p\ ^4G^o$  with



**Figure 3.** Thermal Doppler and Stark widths for Hf II 2642.2 Å spectral line of singly ionized hafnium of multiplet  $6s\ ^4F - 6p\ ^4G^o$  for DA white dwarf atmosphere model with  $T_{eff} = 15000$  K and  $\log g = 8$ , as a function of optical depth  $\tau_{5150}$ .

9 spectral lines, all of which fulfill the conditions of LS-coupling or Russell-Saunders coupling for connection and the selection rules. All labels are taken from Moore (1971); Reader et al. (1980); Ralchenko et al. (2005) and the NIST database, at site <https://www.nist.gov/>.

Taking into account the entire set of calculated spectral lines of singly ionized hafnium, we compared the shift and line width ratios ( $d/W$ ) for each one, which can be seen in Figure 1. This ratio does not depend on the wavelength, temperature or density of the perturber, but it does depend on the quantum numbers and the optical charge seen by the electron. Minimum value of the ratio of 0.197 occurs for the 2259.4 Å line. The ratio increases approximately linearly with the wavelength, up to value 0.307 for the highest wavelength of 3496.8 Å.

For some of the calculated lines of singly ionized hafnium there are relative intensity presented in the NIST database. One of them has relative intensity of 1100 and it is observed at 2642.2 Å, which belongs to the multiplet  $6s\ a^4F_{9/2} - 6p\ z^4G_{11/2}^o$ , as well as the second strongest line of 2938.6 Å, with the relative intensity of 710. The Figure 2 shows the dependence of width on temperature, we notice that as the temperature increases, the width of the line decreases faster - the interval below 40000 K, and more slowly at higher temperatures than this. The spectral line  $6s\ a^4F_{9/2} - 6p\ z^4G_{9/2}^o$  with wavelength is 2938.6 Å from multiplet  $6s\ ^4F - 6p\ ^4G^o$  was compared with the 2642.2 Å line. This results

is in accordance with the Eq. (1), given in the previous section, which provides dependencies from the wavelength and the initial and final energy levels.

In order to determine the influence of the Stark broadening in the atmosphere of hot stars, especially white dwarfs and check its importance and contribution, we tested the spectral line on the DA type model Wickramasinghe (1972) of white dwarfs, with  $T_{eff} = 15000$  K and  $\log g = 8$ . Here, we denoted with  $\tau_{5150}$  optical depth points at the standard wavelength  $\lambda_s = 5150$  Å also used in Wickramasinghe (1972). As can be seen from Figure 3 Stark broadening mechanism is more dominant in comparison with thermal Doppler, starting from the lowest photosphere layer in the atmosphere of the white dwarf. Our result shows 2642.2 Å spectral line from the multiplet  $6s^4F - 6p^4G^0$ .

Insight into Stark broadening parameters is of great importance for the interpretation of spectra of A-type stars and white dwarfs, as we have shown in some of the previous studies Majlinger et al. (2015, 2017); Simić & Sakan (2020); Simić & Sakan (2021). Therefore, the Stark broadening must be taken into consideration when investigating stellar, technological and laboratory plasma.

#### 4. Conclusion

In this paper, the selected transition metal is singly charged hafnium ion, which is detected in atmospheres of metallic stars, as for example the spectrum of CS 22892-052. This element whose energy levels have been determined for 28 known lines provides opportunities for calculation by SMSE method to obtain Stark widths and shifts. We analyzed the shift-to-width ratio for all lines and observed the usual deviations as in previous studies of the transition metal group. We determined the influence of Stark broadening mechanism in the Atmosphere of a DA type white dwarf. The Stark broadening mechanism is very important for white dwarfs atmospheres, and one has to take into account this effect for their investigations, analysis and modeling.

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**Table 1.** Electron-impact (Stark) broadening full widths at half intensity maximum ( $W$ ) and shifts ( $d$ ) for Hf II spectral lines, for a perturber density of  $10^{17}$  cm $^{-3}$  and temperatures from 5000 K to 80000 K.

Transition	T [K]	W [Å]	d [Å]	W [ $10^{12}$ s $^{-1}$ ]	d [ $10^{12}$ s $^{-1}$ ]	$\frac{3kT}{2\Delta E}$
Hf II $a^4F_{3/2} - 6p y^4D_{1/2}^o$ 2381.03 Å	5000	0.103D+00	-0.223D-01	0.344	-0.074	0.124
	10000	0.733D-01	-0.158D-01	0.243	-0.052	0.248
	20000	0.519D-01	-0.111D-01	0.172	-0.037	0.497
	40000	0.367D-01	-0.791D-02	0.121	-0.026	0.993
	80000	0.259D-01	-0.559D-02	0.086	-0.018	1.986
Hf II $6s a^4F_{3/2} - 6p y^4D_{3/2}^o$ 2323.98 Å	5000	0.101D+00	-0.210D-01	0.354	-0.073	0.121
	10000	0.718D-01	-0.148D-01	0.250	-0.051	0.242
	20000	0.508D-01	-0.105D-01	0.177	-0.036	0.485
	40000	0.359D-01	-0.744D-02	0.125	-0.025	0.969
	80000	0.254D-01	-0.526D-02	0.088	-0.018	1.939
Hf II $6s a^4F_{3/2} - 6p y^4D_{5/2}^o$ 2259.39 Å	5000	0.992D-01	-0.196D-01	0.366	-0.072	0.118
	10000	0.701D-01	-0.138D-01	0.259	-0.051	0.236
	20000	0.496D-01	-0.979D-02	0.183	-0.036	0.471
	40000	0.350D-01	-0.692D-02	0.129	-0.025	0.942
	80000	0.248D-01	-0.490D-02	0.091	-0.018	1.885
Hf II $6s a^4F_{3/2} - 6p y^4F_{3/2}^o$ 2572.45 Å	5000	0.112D+00	-0.269D-01	0.318	-0.076	0.134
	10000	0.791D-01	-0.190D-01	0.225	-0.054	0.268
	20000	0.559D-01	-0.134D-01	0.159	-0.038	0.536
	40000	0.395D-01	-0.953D-02	0.112	-0.027	1.073
	80000	0.279D-01	-0.674D-02	0.079	-0.019	2.146
Hf II $6s a^4F_{3/2} - 6p y^4F_{5/2}^o$ 2497.75 Å	5000	0.108D+00	-0.251D-01	0.328	-0.075	0.130
	10000	0.768D-01	-0.177D-01	0.231	-0.053	0.260
	20000	0.543D-01	-0.125D-01	0.164	-0.037	0.521
	40000	0.384D-01	-0.889D-02	0.116	-0.026	1.042
	80000	0.271D-01	-0.628D-02	0.082	-0.018	2.084
Hf II $6s a^4F_{3/2} - 6p z^4G_{5/2}^o$ 3195.12 Å	5000	0.145D+00	-0.439D-01	0.268	-0.081	0.167
	10000	0.102D+00	-0.310D-01	0.189	-0.057	0.333
	20000	0.727D-01	-0.219D-01	0.134	-0.040	0.666
	40000	0.514D-01	-0.155D-01	0.094	-0.028	1.333
	80000	0.363D-01	-0.109D-01	0.067	-0.020	2.665
Hf II $6s a^4F_{5/2} - 6p y^4D_{3/2}^o$ 2394.09 Å	5000	0.108D+00	-0.228D-01	0.357	-0.075	0.125
	10000	0.769D-01	-0.161D-01	0.252	-0.053	0.250
	20000	0.544D-01	-0.114D-01	0.178	-0.037	0.499
	40000	0.384D-01	-0.808D-02	0.126	-0.026	0.999
	80000	0.272D-01	-0.571D-02	0.089	-0.018	1.997
Hf II $6s a^4F_{5/2} - 6p y^4D_{5/2}^o$ 2325.61 Å	5000	0.106D+00	-0.212D-01	0.369	-0.074	0.121
	10000	0.750D-01	-0.150D-01	0.261	-0.052	0.243
	20000	0.530D-01	-0.106D-01	0.184	-0.037	0.485
	40000	0.375D-01	-0.751D-02	0.130	-0.026	0.970
	80000	0.265D-01	-0.531D-02	0.092	-0.018	1.940
Hf II $6s a^4F_{5/2} - 6p y^4D_{7/2}^o$ 2271.39 Å	5000	0.104D+00	-0.199D-01	0.380	-0.072	0.118
	10000	0.736D-01	-0.141D-01	0.268	-0.051	0.237
	20000	0.520D-01	-0.999D-02	0.190	-0.036	0.474
	40000	0.368D-01	-0.706D-02	0.134	-0.025	0.947
	80000	0.260D-01	-0.499D-02	0.095	-0.018	1.895
Hf II $6s a^4F_{5/2} - 6p y^4F_{3/2}^o$ 2658.64 Å	5000	0.120D+00	-0.294D-01	0.322	-0.078	0.139
	10000	0.854D-01	-0.208D-01	0.227	-0.055	0.277
	20000	0.604D-01	-0.147D-01	0.161	-0.039	0.554
	40000	0.427D-01	-0.104D-01	0.113	-0.027	1.109
	80000	0.302D-01	-0.735D-02	0.080	-0.019	2.218



Table 1. Continued

Transition	T [K]	W [Å]	d [Å]	W [10 <sup>12</sup> s <sup>-1</sup> ]	d [10 <sup>12</sup> s <sup>-1</sup> ]	$\frac{3kT}{2\Delta E}$
Hf II 6s a <sup>4</sup> F <sub>5/2</sub> - 6p y <sup>4</sup> F <sub>5/2</sub> <sup>o</sup> 2578.92 Å	5000	0.117D+00	-0.274D-01	0.331	-0.077	0.134
	10000	0.827D-01	-0.193D-01	0.234	-0.054	0.269
	20000	0.585D-01	-0.137D-01	0.165	-0.038	0.538
	40000	0.413D-01	-0.968D-02	0.117	-0.027	1.076
	80000	0.292D-01	-0.684D-02	0.082	-0.019	2.151
Hf II 6s a <sup>4</sup> F <sub>5/2</sub> - 6p z <sup>4</sup> F <sub>7/2</sub> <sup>o</sup> 2531.96 Å	5000	0.114D+00	-0.262D-01	0.337	-0.077	0.132
	10000	0.811D-01	-0.185D-01	0.238	-0.054	0.264
	20000	0.574D-01	-0.131D-01	0.168	-0.038	0.528
	40000	0.406D-01	-0.927D-02	0.119	-0.027	1.056
	80000	0.287D-01	-0.655D-02	0.084	-0.019	2.112
Hf II 6s a <sup>4</sup> F <sub>5/2</sub> - 6p z <sup>4</sup> G <sub>5/2</sub> <sup>o</sup> 3329.17 Å	5000	0.159D+00	-0.486D-01	0.271	-0.082	0.174
	10000	0.113D+00	-0.344D-01	0.192	-0.058	0.347
	20000	0.799D-01	-0.243D-01	0.135	-0.041	0.694
	40000	0.565D-01	-0.172D-01	0.096	-0.029	1.389
	80000	0.399D-01	-0.121D-01	0.067	-0.020	2.777
Hf II 6s a <sup>4</sup> F <sub>5/2</sub> - 6p z <sup>4</sup> G <sub>7/2</sub> <sup>o</sup> 2631.02 Å	5000	0.107D+00	-0.297D-01	0.293	-0.080	0.155
	10000	0.762D-01	-0.210D-01	0.207	-0.057	0.310
	20000	0.539D-01	-0.148D-01	0.146	-0.040	0.621
	40000	0.381D-01	-0.105D-01	0.103	-0.028	1.242
	80000	0.269D-01	-0.743D-02	0.073	-0.020	2.483
Hf II 6s a <sup>4</sup> F <sub>7/2</sub> - 6p y <sup>4</sup> D <sub>5/2</sub> <sup>o</sup> 2406.16 Å	5000	0.114D+00	-0.233D-01	0.373	-0.075	0.125
	10000	0.812D-01	-0.165D-01	0.264	-0.053	0.251
	20000	0.574D-01	-0.116D-01	0.186	-0.037	0.502
	40000	0.406D-01	-0.825D-02	0.132	-0.026	1.004
	80000	0.287D-01	-0.583D-02	0.093	-0.019	2.007
Hf II 6s a <sup>4</sup> F <sub>7/2</sub> - 6p y <sup>4</sup> D <sub>7/2</sub> <sup>o</sup> 2348.17 Å	5000	0.112D+00	-0.219D-01	0.384	-0.074	0.122
	10000	0.795D-01	-0.155D-01	0.271	-0.053	0.245
	20000	0.562D-01	-0.109D-01	0.192	-0.037	0.490
	40000	0.397D-01	-0.775D-02	0.135	-0.026	0.979
	80000	0.281D-01	-0.548D-02	0.096	-0.018	1.959
Hf II 6s a <sup>4</sup> F <sub>7/2</sub> - 6p y <sup>4</sup> F <sub>5/2</sub> <sup>o</sup> 2678.35 Å	5000	0.127D+00	-0.303D-01	0.335	-0.079	0.140
	10000	0.903D-01	-0.214D-01	0.237	-0.056	0.279
	20000	0.638D-01	-0.151D-01	0.167	-0.039	0.559
	40000	0.451D-01	-0.107D-01	0.118	-0.028	1.117
	80000	0.319D-01	-0.757D-02	0.083	-0.019	2.234
Hf II 6s a <sup>4</sup> F <sub>7/2</sub> - 6p y <sup>4</sup> F <sub>7/2</sub> <sup>o</sup> 2269.86 Å	5000	0.933D-01	-0.216D-01	0.341	-0.079	0.137
	10000	0.660D-01	-0.152D-01	0.241	-0.055	0.274
	20000	0.466D-01	-0.108D-01	0.170	-0.039	0.548
	40000	0.330D-01	-0.764D-02	0.120	-0.027	1.096
	80000	0.233D-01	-0.540D-02	0.085	-0.019	2.192
Hf II 6s a <sup>4</sup> F <sub>7/2</sub> - 6p y <sup>4</sup> F <sub>9/2</sub> <sup>o</sup> 2513.79 Å	5000	0.119D+00	-0.260D-01	0.356	-0.077	0.131
	10000	0.845D-01	-0.184D-01	0.252	-0.054	0.262
	20000	0.598D-01	-0.130D-01	0.178	-0.038	0.524
	40000	0.422D-01	-0.920D-02	0.126	-0.027	1.049
	80000	0.299D-01	-0.651D-02	0.089	-0.019	2.097
Hf II 6s a <sup>4</sup> F <sub>7/2</sub> - 6p z <sup>4</sup> G <sub>5/2</sub> <sup>o</sup> 3496.75 Å	5000	0.179D+00	-0.549D-01	0.275	-0.084	0.182
	10000	0.126D+00	-0.388D-01	0.195	-0.059	0.365
	20000	0.895D-01	-0.274D-01	0.137	-0.042	0.729
	40000	0.632D-01	-0.194D-01	0.097	-0.029	1.459
	80000	0.447D-01	-0.137D-01	0.068	-0.021	2.917

Table 1. Continued

Transition	T [K]	W [Å]	d [Å]	W [10 <sup>12</sup> s <sup>-1</sup> ]	d [10 <sup>12</sup> s <sup>-1</sup> ]	$\frac{3kT}{2\Delta E}$
Hf II 6s a <sup>4</sup> F <sub>7/2</sub> - 6p z <sup>4</sup> G <sub>7/2</sub> <sup>o</sup> 3110.01 Å	5000	0.152D+00	-0.425D-01	0.297	-0.082	0.162
	10000	0.108D+00	-0.301D-01	0.210	-0.058	0.324
	20000	0.763D-01	-0.212D-01	0.148	-0.041	0.649
	40000	0.539D-01	-0.150D-01	0.105	-0.029	1.297
	80000	0.381D-01	-0.106D-01	0.074	-0.020	2.594
Hf II 6s a <sup>4</sup> F <sub>7/2</sub> - 6p z <sup>4</sup> G <sub>9/2</sub> <sup>o</sup> 2774.18 Å	5000	0.132D+00	-0.329D-01	0.325	-0.080	0.145
	10000	0.939D-01	-0.232D-01	0.229	-0.056	0.289
	20000	0.664D-01	-0.164D-01	0.162	-0.040	0.579
	40000	0.469D-01	-0.116D-01	0.114	-0.028	1.157
	80000	0.332D-01	-0.822D-02	0.081	-0.020	2.314
Hf II 6s a <sup>4</sup> F <sub>9/2</sub> - 6p y <sup>4</sup> D <sub>7/2</sub> <sup>o</sup> 2464.93 Å	5000	0.125D+00	-0.2512-01	0.389	-0.077	0.129
	10000	0.889D-01	-0.177D-01	0.275	-0.055	0.257
	20000	0.628D-01	-0.125D-01	0.195	-0.038	0.514
	40000	0.444D-01	-0.888D-02	0.137	-0.027	1.028
	80000	0.314D-01	-0.628D-02	0.097	-0.019	2.056
Hf II 6s a <sup>4</sup> F <sub>9/2</sub> - 6p y <sup>4</sup> F <sub>7/2</sub> <sup>o</sup> 2774.83 Å	5000	0.141D+00	-0.335D-01	0.347	-0.081	0.145
	10000	0.100D+00	-0.236D-01	0.245	-0.057	0.289
	20000	0.709D-01	-0.167D-01	0.173	-0.040	0.579
	40000	0.501D-01	-0.118D-01	0.122	-0.028	1.157
	80000	0.354D-01	-0.837D-02	0.086	-0.020	2.315
Hf II 6s a <sup>4</sup> F <sub>9/2</sub> - 6p y <sup>4</sup> F <sub>9/2</sub> <sup>o</sup> 2648.09 Å	5000	0.134D+00	-0.299D-01	0.362	-0.080	0.138
	10000	0.954D-01	-0.212D-01	0.256	-0.056	0.276
	20000	0.674D-01	-0.149D-01	0.181	-0.040	0.552
	40000	0.477D-01	-0.106D-01	0.128	-0.028	1.105
	80000	0.337D-01	-0.749D-02	0.090	-0.020	2.209
Hf II 6s a <sup>4</sup> F <sub>9/2</sub> - 6p z <sup>4</sup> G <sub>7/2</sub> <sup>o</sup> 3318.21 Å	5000	0.177D+00	-0.501D-01	0.303	-0.085	0.173
	10000	0.125D+00	-0.354D-01	0.214	-0.060	0.346
	20000	0.886D-01	-0.250D-01	0.151	-0.042	0.692
	40000	0.626D-01	-0.177D-01	0.107	-0.030	1.384
	80000	0.443D-01	-0.125D-01	0.075	-0.021	2.768
Hf II 6s a <sup>4</sup> F <sub>9/2</sub> - 6p z <sup>4</sup> G <sub>9/2</sub> <sup>o</sup> 2938.64 Å	5000	0.151D+00	-0.382D-01	0.330	-0.083	0.153
	10000	0.107D+00	-0.270D-01	0.234	-0.059	0.306
	20000	0.758D-01	-0.191D-01	0.165	-0.041	0.613
	40000	0.536D-01	-0.135D-01	0.117	-0.029	1.226
	80000	0.379D-01	-0.956D-02	0.082	-0.020	2.451
Hf II 6s a <sup>4</sup> F <sub>9/2</sub> - 6p z <sup>4</sup> G <sub>11/2</sub> <sup>o</sup> 2642.20 Å	5000	0.134D+00	-0.298D-01	0.363	-0.080	0.138
	10000	0.951D-01	-0.210D-01	0.256	-0.056	0.276
	20000	0.673D-01	-0.149D-01	0.181	-0.040	0.551
	40000	0.475D-01	-0.105D-01	0.128	-0.028	1.102
	80000	0.336D-01	-0.745D-02	0.090	-0.020	2.204

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