A GEOMETRICAL CHART OF ALTERED TEMPORALITY (AND SPATIALITY)

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Abstract: The paper presents, to our knowledge, a first fairly comprehensive and mathematically well-underpinned classification of the psychopathology of time (and space). After reviewing the most illustrative first-person accounts of "anomalous/peculiar" experiences of time (and, to a lesser degree, space) we introduce and describe in detail their algebraic geometrical model. The model features six qualitatively different types of the internal structure of time dimension and four types of that of space. As for time, the most pronounced are the ordinary "past-present-future," "present-only" ("eternal/everlasting now") and "no-present" (time "standing still") patterns. Concerning space, the most elementary are the ordinary, i.e., "here-and-there," mode and the "here-only" one ("omnipresence"). We then show what the admissible combinations of temporal and spatial psycho-patterns are and give a rigorous algebraic geometrical classification of them. The predictive power of the model is illustrated by the phenomenon of psychological time-reversal and the experiential difference between time and space. The paper ends with a brief account of some epistemological/ontological questions stemming from the approach.

Keywords: Mental Space-Times – Pencil of Conics/Lines – Cremona Transformations

1 Introduction

Time — "the supreme law of nature" after Sir Arthur Stanley Eddington, a world-famous astrophysicist of the last century — is undoubtedly one of the deepest mysteries science has ever faced. Indeed, one would hardly find something that is, on the one hand, so intimately connected with our experience and yet, on the other, so difficult to come to grips with. Nothing, perhaps, can better illustrate this point than a large group of phenomena that are collectively referred to as the *psychopathology* of time, that is, all "anomalous/peculiar" experiences of time as invariably encountered and reported in various mental psychoses, drug-induced states, deep meditative and mystical states as well as in many other "altered" states of consciousness [1–7]. For such peculiar fabric of psychological time comprises, as we shall see in more detail, such bizarre, paradoxical and mind-boggling forms as "eternity, everlasting now," "arrested/suspended" time, time "going backward," and even "disordered/fragmented" time, to mention the most pronounced of them.

Up to date, there exists no acceptable psychological/neurological model capable of properly dealing with these fascinating time constructs and underpinning any logical classification of them. The reason why this is so rests, in our opinion, upon the following two facts. First, these extraordinary experiences of time (and, of course, space as well) are inherently participatory, non-reproducible and subjective and, so, seriously at odds with current methodologies/paradigms of science, which strive for reproducibility and objectivity. Second, the most pronounced departures from the "consensus" reality are so foreign to our "waking" mind that their properties defy our common sense logic and cannot be adequately communicated in words; an interested scholar has to go through a large number of relevant first-hand accounts/narratives and acquire the ability to read between the lines in order to spot an(y) underlying conceptual pattern. We are therefore convinced that further progress in our understanding of these phenomena will inevitably entail a serious shift in the corresponding scientific paradigms to reveal their true epistemological/ontological status and be accompanied by the increasing use of sufficiently abstract mathematical concepts to properly grasp their qualitative properties.

Our study of psychopathological (space-)times has, from the very beginning, been pursued in accordance with this strategy [7–9]. The model discussed in the second part of the paper thus features not only a fairly high level of abstraction, but it also poses a serious challenge to some generally accepted dogmas in natural sciences. Formally, it employs advanced geometrical concepts, like a projective space and/or Cremona transformations. Conceptually, it relies on a daring and far-reaching assumption that the anecdotal, first-person descriptions of extraordinary states of consciousness are on a par with standard observational/experimental evidence in natural sciences. It is this "abstract geometrization of the first-person perspective" that gives our approach a remarkable unifying and predictive power and makes it a very promising conceptual step towards the ultimate unveiling of the riddle of time. The purpose of the paper is to demonstrate this. The presentation is focussed on conceptual issues rather than mathematical technicalities, the latter being reduced to the extent that also the reader with a comparatively slight mathematical background can easily follow the main line of reasoning.

2 Mental (Space-)Times: Most Illustrative Cases

We shall start with a compact, yet comprehensive enough, review of the most distinguished forms of "anomalous" experience of time. This review is unique in that it consists solely of first-person accounts/narratives, three or four per each mode. This way even the uninitiated reader can get a fairly clear idea about the nature of experiences involved and realize the source and character of possible difficulties one is likely to face when attempting to mathematically model these experiences.

"Eternity," alias "eternal/everlasting now" 2.1

This is perhaps the most pronounced and in the literature best-documented kind of profoundly "distorted" sense of time. It is a sort of compressing, telescoping of past, present and future into the present moment that is experienced as "eternal/everlasting now." One of the best portrayals of what this experience looks like is found in the following account [10, p. 46]:

I woke up in a whole different world in which the puzzle of the world was solved extremely easily in a form of a different space. I was amazed at the wonder of this different space and this amazement concealed my judgement, this space is totally distinct from the one we all know. It had different dimensions, everything contained everything else. I was this space and this space was me. The outer space was part of this space, I was in the outer space and the outer space was in me...

Anyway, I didn't experience time, time of the outer space and aeons until the second phase of this dream. In the cosmic flow of time you saw worlds coming into existence, blooming like flowers, actually existing and then disappearing. It was an endless game. If you looked back into the past, you saw aeons, if you looked forward into the future there were aeons stretching into the eternity, and this eternity was contained in the point of the present. One was situated in a state of being in which the "will-be" and the "vanishing" were already included, and this "being" was my consciousness. It contained it all. This "beingcontained" was presented very vividly in a geometric way in form of circles of different size which again were all part of a unity since all of the circles formed exactly one circle. The biggest circle was part of the smallest one and vice versa...

This narrative is remarkable in a couple of aspects. Not only does the subject try to understand his uncanny experience of time in terms of a simple geometrical model, but he also pays particular attention to the spatial fabric of his extraordinary state, which also differs utterly from what is regarded

as a normal/ordinary perception of space; in fact, the subject finds himself to be one/fused with space!

Another description of the same kind of mental space-time structure is taken from Atwater [11, Chpt. 2]. It is based on one of many author's near-death experiences, which was also accompanied by a fascinating archetypal imaginery:

This time, I moved, not my environment, and I moved rapidly.... My speed accelerated until I noticed a wide but thin-edged expanse of bright light ahead, like a "parting" in space or a "lip," with a brightness so brilliant it was beyond light yet I could look upon it without pain or discomfort... The closer I came the larger the parting in space appeared until... I was absorbed by it as if engulfed by a force field... Further movement on my part ceased because of the shock of what happened next. Before me there loomed two gigantic, impossibly huge masses spinning at great speed, looking for all the world like cyclones. One was inverted over the other, forming an hourglass shape, but where the spouts should have touched there was instead incredible rays of power shooting out in all directions... I stared at the spectacle before me in disbelief... As I stared, I came to recognize my former Phyllis self in the midupperleft of the top cyclone. Even though only a speck, I could see my Phyllis clearly, and superimposed over her were all her past lives and all her future lives happening at the same time in the same place as her present life. Everything was happening at once! Around Phyllis was everyone else she had known and around them many others... The same phenomenon was happening to each and all. Past, present, and future were not separated but, instead, interpenetrated like a multiple hologram combined with its own reflection. The only physical movement anyone or anything made was to contract and expand. There was no up or down, right or left, forward or backward. There was only in and out, like breathing, like the universe and all creation were breathing — inhale/exhale, contraction/expansion, in/out, off/on.

The last example, but by no means less astounding than the former two, is borrowed from Braud [12] and depicts in great detail and clarity a gradual transformation of our ordinary, waking sense of time (and space) into that of "eternity" (and "omnipresence"):

I get up and walk to the kitchen, thinking about what a timeless experience would be like. I direct my attention to everything that is happening at the present moment — what is happening here, locally, inside of me and near me, but non-locally as well, at ever increasing distances from me. I am imagining everything that is going on in a slice of the present — throughout the country, the planet, the universe. It's all happening at once.

I begin to collapse time, expanding the slice of the present, filling it with what has occurred in the immediate "past." I call my attention to what I just did and experienced, what led up to this moment, locally, but keep these events within a slowly expanding present moment. The present slice of time slowly enlarges, encompassing, still holding, what has gone just before, locally, but increasingly non-locally as well. By now, I am standing near the kitchen sink. The present moment continues to grow, expand. Now it expands into the "future" as well. Events are gradually piling up in this increasingly larger moment. What began as a thin, moving slice of time, is becoming thicker and thicker, increasingly filled with events from the "present," "past," and "future." The moving window of the present becomes wider and wider, and moves increasingly outwardly in two temporal directions at once. It is as though things are piling up in an ever-widening present.

The "now" is becoming very thick and crowded! "Past" events do not fall away and cease to be; rather, they continue and occupy this ever-widening present. "Future" events already are, and they, too, are filling this increasingly thick and full present moment. The moment continues to grow, expand, fill, until it contains all things, all events. It is so full, so crowded, so thick, that everything begins to blend together. Distinctions blur. Boundaries melt away. Everything becomes increasingly homogeneous, like an infinite expanse of gelatine. My own boundaries dissolve. My individuality melts away. The moment is so full that there no longer are separate things. There is no-thing here. There are no distinctions.

A very strong emotion overtakes me. Tears of wonder-joy fill my eyes. This is a profoundly moving experience. Somehow, I have moved away from the sink and am now several feet away, facing in the opposite direction, standing near the dining room table. I am out of time and in an eternal present. In this present is everything and no-thing. I, myself, am no longer here. Images fade away. Words and thoughts fade away. Awareness remains, but it is a different sort of awareness. Since distinctions have vanished, there is nothing to know and no one to do the knowing. "I" am no longer localized, but no longer "conscious" in the usual sense. There is no-thing to be witnessed, and yet there is still a witnesser.

The experience begins to fade. I am "myself" again. I am profoundly moved. I feel awe and great gratitude for this experience with which I have been blessed...

2.2 Time "standing still," alias "arrested/suspended" time

Another well-documented and quite abundant anomalous temporal mode. A couple of examples are found in Tellenbach [13, p. 13]:

I sure do notice the passing of time but couldn't experience it. I know that tomorrow will be another day again but don't feel it approaching. I can estimate the past in terms of years but I don't have any connection to it anymore. The time standstill is infinite, I live in a constant eternity. I see the clocks turn but for me time does not flow... Everything lies in one line, there are no differences of depth anymore... Everything is like a firm plane...

and [*ibid*, p. 14]

Everything is very different in my case, time is passing very slowly. Nights last so long, one hour is as long as usually a whole day...Sometimes time had totally stood still, it would have been horrifying. Even space had changed: Everything is so empty and dark, everything is so far away from me...

I don't see space as usual, I see everything as if it were just a background. It all seems to me like a wall, everything is flat. Everything presses down, everything looks away from me and laughs...

Both reports are given by depressive (melancholic) patients. It is worth noticing here that when time comes to a stillstand, perceived space seems to lose one dimension, becoming thus two-dimensional. A slightly more detailed description of this time pattern we succeeded in finding in a paper by Muscatello and Giovanardi Rossi [14, p. 784]:

Time is standing still for me, I believe. It is perhaps only a few moments that I have been so bad. I look at a clock and I have the impression, if I look at it again, that an enormous period of time has passed, as if hours would have passed instead only a few minutes. It seems to me that a duration of time is enormous. Time does not pass any longer, I look at the clock but its hands are always at the same position, they no longer move, they no longer go on; then I check if the clock came to a halt, I see that it works, but the hands are standing still. I do not think about my past, I remember it but I do not think about it too much. When I am so bad, I never think about my past. Nothing enters my mind, nothing... I did not manage to think about anything. I did not manage to see anything in my future. The present does not exist for me when I am so bad...the past does not exist, the future does not exist.

The following vignette is taken form a treatise on mescaline-induced experimental psychoses by Beringer [15, p. 311]:

The strangest thing was that every once in a while my normal time-awareness, as far as these figures were concerned, got totally lost; time was no longer a stream, which flew away and whose flux could have been measured, but it was rather similar to a sea, which as a whole stood still and which was in itself only a chaotic and utter jumble. I was no longer able to understand the continuous becoming of the figures as a sequence in a certain time direction, but sometimes the colours and forms flew into an indescribable jumble, as if the previously alternating figures were now experienced all simultaneously. Had I previously seen these figures in a constant motion, so now it was only a colorful and inexpressible manifold there in which I was not able to perceive any motion anymore. When I totally sank into the show of the figures, it happened every now and then that I also sank into this time-still-standing, where the succession was transformed into a still standing present. Not only am I now not able to formulate these interruptions of the normal time experience, I am also almost unable to imagine my experience of them any more. When I tore myself away from these figures and violently turned myself to the outer world, this anomalous time experience was no longer here, but this disturbance of the sense of time found its expression in a form of illusion that an immense long time must have passed since my last waking-up.

2.3 Time "going/flowing backward"

This kind of time pathology is very often found in mental psychoses [1,2,5,16–18]. Here is a representative case, communicated by a schizophrenic patient of Fischer [19, p. 556]:

Yesterday at noon, when the meal was being served, I looked at the clock: why did no one else? But there was something strange about it. For the clock did not help me any more and did not have anything to say to me any more. How was I going to relate to the clock? I felt as if I had been put back, as if something of the past returned, so to speak, toward me, as if I were going on a journey. It was as if at 11:30 a.m. it was 11:00 a.m. again, but not only time repeated itself again, but all that had happened for me during that time as well. In fact, all of this is much too profound for me to express. In the middle of all this something happened which did not seem to belong here. Suddenly, it was not only 11:00 a.m. again, but a time which passed a long time before was there and there inside — have I already told you about a nut in a great, hard shell? It was like that again: in the middle of time I was coming from the past towards myself. It was dreadful. I told myself that perhaps the clock had been set back, the orderlies wanted to play a stupid trick with the clock. I tried to envisage time as usual, but I could not do it; and then came a feeling of horrible expectation that I could be sucked up into the past, or that the past would overcome me and flow over me. It was disquieting that someone could play with time like that, somewhat daemonic...

A brief and concise description of "psycho-time-reversal" is found in Laing [20, p. 148]:

I got the impression that time was flowing backward; I felt that time proceeded in the opposite direction, I had just this extraordinary sensation, indeed... the most important sensation at that moment was, time in the opposite direction.... The perception was so real that I looked at a clock and, I do not know how, I had the impression that the clock confirmed this feeling, although I was not able to discern the motion of its hands....

A similar depiction is also furnished by a depressive patient of Kloos [21, p. 237]:

As I suddenly broke down I had this feeling inside me that time had completely flown away. After those three weeks in a sickcamp, I had this feeling that the clock hands run idle, that they do not have any hold. This was my sudden feeling. I did not find, so to speak, any hold of a clock and of life anymore, I experienced a dreadful psychological breakdown. I do not know the reason why I especially became conscious of the clock. At the same time, I had this feeling that the clock hands run backward... There is only one piece left, so to speak, and that stands still. I could not believe that time really did advance, and that is why I thought that the clock hands did not have any hold and ran idle... As I worked and worked again, and worried and did not manage anything, I simply had this feeling that everything around us (including us) goes back... In my sickness I simply did not come along and then I had this delusion inside me that time runs backward...I did not know what was what anymore, and I always thought that I was losing my mind. I always thought that the clock hands run the wrong way round, that they are without any meaning. I just stood-up in the sickcamp and looked at the clock — and it came to me then at once: well, what is this, time runs the wrong way round?!...I saw, of course, that the hands moved forward, but, as I could not believe it, I kept thinking that in reality the clock runs backward...

2.4 "Disordered/fragmented" time

The following experience, voluntarily induced by mescaline, is the most representative one we have been able to find in the literature available [22, p. 295]:

For half an hour nothing happened. Then I began feeling sick; and various nerves and muscles started twitching unpleasantly.

Then, as this wore off, my body became more or less anaesthetized, and I became "de-personalized," i.e., I felt completely detached from my body and the world...

This experience alone would have fully justified the entire experiment for me..., but at about 1.30 all interest in these visual phenomena was abruptly swept aside when I found that time was behaving even more strangely than color. Though perfectly rational and wide-awake...I was not experiencing events in the normal sequence of time. I was experiencing the events of 3.30 before the events of 3.0; the events of 2.0 after the events of 2.45, and so on. Several events I experienced with an equal degree of reality more than once. I am not suggesting, of course, that the events of 3.30 happened before the events of 3.0, or that any event happened more than once. All I am saying is that I experienced them, not in the familiar sequence of clock time, but in a different, apparently capricious sequence which was outside my control.

By "I" in this context I mean, of course, my disembodied self, and by "experienced" I mean learned by a special kind of awareness which seemed to comprehend yet be different from seeing, hearing, etc.... I count this experience, which occurred when, as I say, I was wide awake and intelligent, sitting in my own armchair at home, as the most astounding and thought-provoking of my life....

And here is another mescaline-borne episode of a very similar time's sense [15, p. 148]:

While walking upstairs, a sudden and as if nailed-down picture of this moment, the momentary view of Dr. M., Dr. St. and myself in space, attracted my attention. This repeated itself on different stairs. At the top of the stairway there seemed to be no continuity of time at all, the whole course of events was only a mess of separate situations without any connection. And these situations, in case of active work, could later have been connected in the same way in which one can observe a celluloid film. Yet at the same time these situations — in both experiencing and a direct reproduction of the happening afterwards — carried the character of the independent and disconnected. A

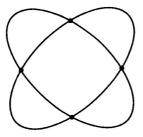
strange next-to-each-other-ness, not a one-after-the-other-ness; they have no position in time, time has no sense here...

From these first-hand accounts it is quite obvious that the fabric of psychological time is so intricate, complex and multifarious that, at first sight, it may seem to lie completely beyond grasp of any mathematical framework. Yet, the contrary is true. In what follows we shall introduce and describe in detail a simple algebraic geometrical model that not only is capable of qualitatively accounting for all the "non-ordinary" time structures mentioned above, but also predicts some novel forms of these.

Pencil Dimensions of Time/Space and Their Mental Counterparts

3.1 Time dimension viewed as a pencil of conics and its (mental) patterns

A cornerstone of our model of the perceived time dimension is conics and their simplest, i.e., linear and single parametrical, aggregates, usually called pencils [23–25]. A conic is an algebraic curve analytically defined by a second order (i.e., quadratic) equation. It is composite (singular) or proper according as this equation is factorable or not. A hyperbola, a parabola and an ellipse are all familiar (and the only) examples of proper conics (with a non-empty image). A composite conic consists of either a pair of (straight-)lines, which can be distinct or coincident, or of a single point. Two distinct conics lying in the same plane have, in general, four points in common (see Fig. 1, left); these, of course, need not be all distinct and/or real. Any two coplanar conics define a unique pencil of conics, viz. the totality of conics that pass through each of the points shared by the two conics (see Fig. 1, right); these common points are called base points of the pencil. Any pencil of conics in the real plane contains at least one composite conic, and maximum three (not necessarily distinct and/or of the same type). Given a pencil of conics, a point of the plane that is not a base point of the pencil lies on exactly one (possibly composite) conic of the pencil, henceforth called the "on-conic." The remaining proper conics of the pencil ("off-conics") are found to form two different, infinitely large disjoint families: one family comprises those proper conics which have the point (henceforth the "reference point") in their interior ("in-conics"), while the other features those conics which have this reference point in their exterior ("ex-conics"). For a given proper conic, a point, not on the conic, is its exterior or interior point depending on whether or not it lies on a line



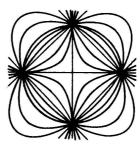
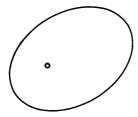


Figure 1. Two distinct conics (ellipses) in the real plane (left) define a unique pencil (right). This pencil, which is the most general one, features four distinct base points and three distinct composite conics (each being a pair of distinct lines). Out of its infinite number of proper conics only nine are shown: four ellipses, one circle and four hyperbolas.

tangent to the conic (see Fig. 2); the exterior/interior of the conic is thus the set of all its exterior/interior points.



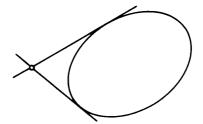


Figure 2. An interior (*left*) and exterior (*right*) point (small circle) of a given proper conic (drawn as an ellipse); the right hand side of the figure also illustrates existence of (two distinct) tangent lines issued from the point to the conic. If the point in question is regarded as the reference point, then the conic on the lef-hand side is an in-conic, whereas that on the right-hand side — an ex-conic.

Why do we pay so much attention, and ascribe so much importance, to this configuration? Simply because in the case when the reference point falls on a proper conic this configuration lends itself as an enunciation of our ordinary experience/sense of time (dimension). To spot this correspondence [26] we take the reference point as a representation of the observer/subject and conceive each conic of the pencil as a single event/moment of time, with the understanding that the ex-conics represent events of the past, the inconics stand for events of the future, and that the unique on-conic answers, naturally, to nothing but the present moment, the "now" — as depicted in

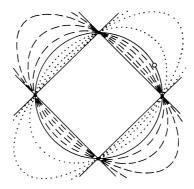


Figure 3. A non-trivial structure within a pencil of conics induced by a generic position of the reference point (small circle) and qualitatively reproducing our common perception of time. The single proper on-conic (solid curve, the present moment) separates the proper off-conics into two distinct domains: the domain of the in-conics (dotted, events of the future) and that of the ex-conics (dashed, events of the past). In both the cases, only a few conics are drawn.

Fig. 3. It is important for the reader to realize at this point a fundamental difference between the conventional, physical concept of time dimension and that of ours. While the former portrays time, loosely speaking, as a line and labels events by points on this line, our theory regards time as a nontrivial geometrical configuration consisting of a given point and an infinite collection of conics, each event being represented by a pair comprising the very point and a conic of the set. In other words, in our model an event/moment of time, rather than being a structureless element/point, possesses itself an *intrinsic* geometrical structure, in virtue of which we are able to introduce a qualitative distinction between individual events (or, better, groups thereof). And our next task is to show that this distinction is very sensitive to the position of the reference point with respect to the distinguished objects of the set. And, indeed, apart from the "pastpresent-future" pattern, our model gives rise to other two prominent, in a sense dual to each other, structures. These correspond, as the reader may have noticed, to the cases where the reference point coincides with a base point of the pencil (Fig. 4b), or falls on one of its composite conics (Fig. 4c). In the former case, clearly, all the proper conics are on-conics, whereas in the latter case the pattern is lacking any such conic, being endowed with ex- and in-conics only. Hence, the corresponding time dimension, in the former case, consists solely of the present moments (the "present-only" mode), whilst, in the latter case, it comprises only the past and future, being devoid

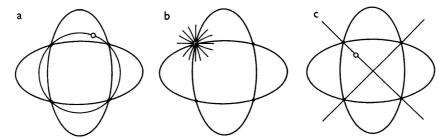


Figure 4. The three qualitatively different patterns of the pencil-borne temporal dimension according as the reference point (a small circle) is (a) incident with a proper conic (see Fig. 3), (b) coincides with one of the base points, or (c) falls on a composite conic of the generating pencil. In the case b, out of an infinity of conics incident with the reference point only (the segments of) a few of them are illustrated. The pencil is, as in Fig. 1, of the most general type.

of the proper moment of the present (the "no-present" mode). Let us try to rephrase these two unusual temporal arrangements in terms of pathological temporal constructs listed in the previous section. We readily find out that the present-only pattern accounts for nothing but experiences of "eternity," "everlasting now." The no-present design is seen to be a proper fit for the time "standing still" mode; for our feeling that time "flows," "proceeds" is unequivocally tied to the notion of the present moment, the "now," as the linking element between the past and future and so it is only too natural to assume that the absence of this element in the pattern should correspond to a complete suspension/cessation of the (sense of) time's flow.

At this point, it is instructive to make a slight digression and discuss a very interesting feature of our approach that has a serious bearing on the very meaning of the term "pathological" when it comes to the concept of time. This feature tells us about a relative probability of the occurrence of the above-discussed three patterns of time in the realm of psychopathology. This probability should not be understood in a strict sense of the word, but rather in a looser, algebraic geometrical sense. The reasoning goes as follows. The conics of any pencil sweep up the whole plane and as the latter contains ∞^2 (double infinity) of points, there are ∞^2 of potential past-present-future patterns. Next, as our pencil features three composite conics, each of these is a pair of distinct lines, and a line possesses ∞^1 (single infinity) of points, we have $3 \times 2 \times \infty^1 = 6 \times \infty^1 \approx \infty^1$ of no-present modes. And, finally, as our pencil features four base points, there are just four present-only structures. We see a clear predominance of the past-

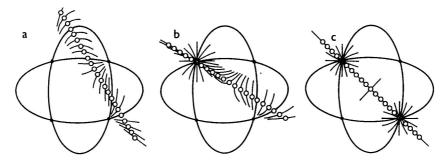


Figure 5. The three distinct types of "line-related" pencil-patterns of time in dependence on whether the reference line is incident with (a) zero, (b) one, or (c) two base points of the pencil. In each case, every illustrated point (a small circle) of the line is accompanied by a drawing of a small portion of the conic incident with this particular point.

present-future mode within the group; no wonder that it corresponds to our "ordinary," "consensus" experience of time. Equivalently, this explains why experiences of "eternity" and/or time "standing still" are regarded/referred to as "anomalous/peculiar;" for the relative probability of their occurrence with respect to our "ordinary" experience of time is truly negligible.

Let us examine next the other conceivable forms of generic pencil-time. We shall assume that instead of a single reference point there is a whole infinity of them, and these are, for simplicity, taken to form a line. What different kinds of time dimension do we find in this case? Remarkably, there are, like in the previous case, three of them. They differ from each other, as depicted in Fig. 5, in the position of this line with respect to the base points of the generating pencil of conics, being in the sequel labelled, respectively, as a zero-, one- and two-point pattern according as the reference line hits no, one or two of the base points. Obviously, these line-related temporal structures can each be regarded as composed of an infinite number of basic, point-related patterns. This composition reads:

type	past-present-future	present-only	no-present
zero-point	infinity	none	six
one-point	infinity	one	three
two-point	none	two	infinity

The numbers in the first two columns are readily discernible from Fig. 5 and the definition of the corresponding elementary modes. It is only the last (no-present) column that requires a word of explanation. Thus, the number in the first line (six) is the number of intersections of the reference line with the composite conics of the pencil; it follows from the facts that our pencil features three composite conics, each of these is a pair of distinct lines, and in a projective plane every line is incident with any other line. The number in the second line (three) answers to the fact that if the reference line passes through a single base point, the latter absorbs three of these six points. Finally, when the reference line joins two base points, it becomes a component of a composite conic, i.e., every point of it lies on the composite conic in question. We further see that, among the composites, only one, the one-point mode (Fig. 5b), features all the three types of elementary patterns, and, similarly, only one, the two-point mode (Fig. 5c), lacks the most familiar of them. On the other hand, there is only one elementary pattern, the no-present one, that enters all the three kinds of composites, and only one, the present-only mode, whose number is always finite. It is very intriguing to see that there is no homogeneous composite.

What are the phenomenological counterparts of these composite temporal patterns? Clearly, each of them must be a mixture/superposition of the time's experiences we have found to correspond to the elementary patterns involved. And these are strange constructs indeed. Thus, the zeropoint mode corresponds to such an uncanny state of consciousness where the subject encounters an infinite tangle of "ordinary" experiences of time, differing from each other in the location of the moment of the present and, consequently, in the spans of the regions of past/future, this perception being accompanied by the sense of time "standing still." The one-point case is even more complex, as it includes, on top of the above, also the feeling of "eternity." And these experiences are very much like those of "disordered/fragmented" time given in the previous section! But what about the two-point structure, an intricate blend of the sense of "eternity/everlasting now" and that of time "standing still"? This kind of experience was privately communicated to one of us by Linda Howe [27], an instructor in the "akashic records" technique:

One common scenario is when the sense of the self is so expanded, beyond any physical boundary.... In this aspect, the awareness of being one with, or a part of, all that is can be profound. The illusion of separation can be perceived as dissolving and, at the same time, the awareness of the oneness, or unity,...becomes heightened, sometimes acutely so... In this the experience of time is dramatic in its expansion and simultaneous contraction. There is a sense that there is only one moment, that all of time/eternity is held in that instant, very compressed and as powerful as one's imagination can conceive.

Simultaneously, there is a sense that there is no time in the expansion. That all is holding still. Not even slow motion, but no motion. A total suspension of time is experienced. This is the all time/no time paradox.

There remains only one mode to be explained, namely the experience/sense of time "going/flowing backward." To this end, we shall return to our ordinary, past-present-future pattern (Fig. 4a) and examine what happens to this pattern as the reference point starts "moving" away from its original position. This "motion," as delineated in Figs. 6a-c, is assumed to take place in such a way that the point always remains incident with one and the same conic. As it is quite obvious from this figure, the qualitative structure of the original pattern (Fig. 6a) is preserved until the reference point, en route, hits a base point (Fig. 6b), in which case the pattern acquires its present-only type. Further motion of the reference point clearly leads to re-establishment of the original type, but with one remarkable difference - with the in-conics and ex-conics having swapped their roles (Fig. 6c)! This means nothing but that the time's arrows generated by the two past-present-future patterns, although sharing the same present moment, point in the opposite directions! One could hardly find a more elementary explanation of time-reversal.

3.2 Space dimension viewed as a pencil of lines and its (mental) patterns

It is evident that that the concept of a pencil, with conics as its constituting elements, turns out to be an extremely fertile framework for getting a

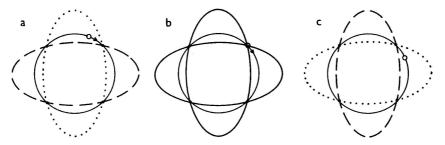
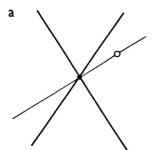


Figure 6. An elementary explanation of the phenomenon of a psychological time-reversal in terms of our pencil-borne model of time dimension. As in Fig. 3, the heavy curve(s) is (are) the on-conic(s), while those drawn as dotted/dashed represent the in-/ex-conics. A little arrow indicates the direction of the motion of the reference point (a small circle).

deeper qualitative insight into the fine structure of psychological time dimension. Motivated by this finding, it is only too natural to address also the structure of psychological space in a similar fashion, i.e., retaining the concept, replacing only its constituting elements. As to our senses, space appears to have a less complex structure than time, and a line is a simpler geometrical object than a proper conic; we shall take a spatial dimension to be represented by a pencil of lines [8,9,25,26,28]. Our reasoning will parallel that of the previous (sub)section, which will enable us to see how our approach gets to grips with the fundamental difference between time and space at the perceptual level. Two distinct lines in a plane suffice to define a unique pencil, i.e., the set of all lines of the plane that pass through the point shared by the two (the point in question being called the vertex of the pencil). As any two lines in a projective plane have always one, and only one, point in common, there exists only one type of a pencil of lines; this is the first fundamental difference from the case of conics. Given a pencil of lines, a point of the plane (the reference point) that is different from the vertex of the pencil is incident with exactly one line of the pencil (Fig. 7a); this line will henceforth be called the on-line, the remaining lines of the pencil being termed off-lines. This particular a-pencil-of-lines-and-a-point configuration qualitatively mimics our "ordinary," "here-and-there" sense of space, with the on-line standing for "here" and off-lines for "there." It is a spatial counterpart of the "ordinary," past-present-future pattern of time (Fig. 4a). However, it must be pointed out here that, unlike off-conics, off-lines have all the same footing with respect to the reference point; this feature thus serves as a nice explanation why, in our "ordinary" state of consciousness, perceived space has a rather trivial structure when compared to that of perceived time. Another point-related spatial pattern is the one where the reference point is identical with the vertex of the pencil (Fig. 7b); as now all the lines of the pencils are on-lines, we get the "here-only" structure. Being a twin of the "eternity," "everlasting now" mode (Fig. 4b), this structure must necessarily be inherent to those "non-ordinary" states of consciousness that are characterized by feelings of "omnipresence," or "fusion/oneness" with the universe. These here-and-there and here-only modes are obviously the only elementary patterns of pencil-space; for a line is so simple an object that there exist no singular forms of it and, so, there does not exist any spatial analogue of the no-present pattern. And as there are ∞^2 potential here-and-there modes, but just a single here-only one, it is only natural that it is the former that underlies our "consensus" perception of space.



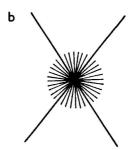
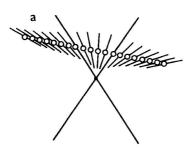


Figure 7. The two qualitatively different elementary patterns of the pencil-borne space dimension depending on whether the reference point is different (a) or not (b) from the vertex of the generating pencil of lines; in the latter case, out of an infinity of lines passing through the reference point only (the segments of) several of them are shown. Compare with Figures 4a and 4b, respectively.



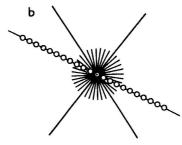


Figure 8. The two composite modes of space pencil-borne dimension, differing from each other in the position of the reference line (illustrated as a range of small circles) with respect to the vertex of the generating pencil of lines. Similarly to Fig. 5, every illustrated reference point goes with a drawing of a small part of the line incident with this particular point.

The cases with the reference line are also structurally simpler than those of time dimension. There are, as the analogy suggests, a couple of them according as the line avoids the vertex (Fig. 8a) or is incident with it (Fig. 8b). As it can easily be recognized from Fig. 8, the former case is a compound of a single infinity of sole here-and-there modes, whereas the latter features a combination of both the elementary modes, with the preponderance of the more familiar of them. Accordingly, a subject experiencing the "avoiding-vertex" mode feels to be localized at every point ("multipresent") along the particular space dimension, while that in a state backed by the "hitting-vertex" mode should feel to be both localized at a particular position of and simultaneously stretched out along the dimension in question.

4 Pencil-Borne Space-Time and the Varieties of its Internal Structure

So far we have treated time and space as two completely unrelated dimensions, which is of course in marked contrast to how the two aspects of reality are perceived to exist. Moreover, we have dealt with a single space dimension only, while our senses tell us that there are (at least) three of them. So we have to refine our model accordingly to comply with these constraints.

To furnish this task, it is necessary to move from the (projective) plane into the (projective) space and — following and extending our recent work [26,28] — consider a specific geometrical configuration comprising three distinct, non-coplanar pencils of lines (generating spatial dimensions) and a single pencil of conics (time). The planes carrying the pencils of lines are taken to be collinear, i.e., having a line in common, and none of the vertices of the pencils (denoted as B_i , i=1,2,3) is assumed to lie on this shared line $(\mathcal{L}^{\mathrm{B}})$. The pencil of conics is, naturally, situated in the plane defined by the three vertices (as these are assumed not to lie on a line), and its base points are these vertices and the point (L) of incidence of the plane and the line \mathcal{L}^{B} , as portrayed in Fig. 9. The reader may get an impression that our option for this configuration is completely arbitrary. This is, however, not the case, for this configuration plays a prominent role in the theory of socalled Cremona transformations between two projective spaces of dimension three.^a For what follows it suffices that the reader shares our intuitive belief that there is indeed something special to the above-described four-pencil configuration so that Nature found it worth making use of [26,28]. It is evident that this remarkable configuration, as it stands, can represent only a bare space-time, i.e., the space-time devoid of any subject/observer. So, in order to introduce the latter into our model, the configuration has to be endowed with an additional geometrical object. This can, of course, be done in a number of ways, one of the simplest being in terms of a single line (denoted as \mathcal{L}^* in Fig. 9). Armed with this premise and the postulates and findings of the previous section, we are able to find out which kinds of spatial and temporal patterns discussed above are mutually compatible (i.e., can form and "live together" on a single manifold) and thereby arrive at a first fairly comprehensive and mathematically well-underpinned classification of the psychopathology of time and space. It is not hard to see that this task

^aA proper explanation of what a Cremona transformation is and what kind(s) of distinguished structures it entails would, however, take us too far afield from the main topic of this paper: the interested reader is therefore referred to consult our above-mentioned papers [26,28] and/or a — though for first reading a bit difficult — book by Hudson [29].

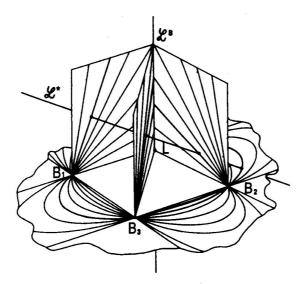


Figure 9. A particular geometrical configuration comprising three pencils of lines and a single pencil of conics, the latter being located in the plane defined by the vertices of the pencils of lines. Out of an infinity of lines, only several are drawn in each of the three pencils; similarly, only a few proper conics (all being ellipses) are shown in the pencil of conics. The symbols are explained in the text.

simply boils down to examining all possible positions of the reference line \mathcal{L}^* within this configuration that lead to qualitatively different arrangements of pencil-patterns induced by the point(s) of intersection of the line with the four pencil-carrying planes.

We shall, of course, start with the case when the reference line is in a generic position with respect to the four planes. As it is obvious from Fig. 9, in this case the line cuts each of these planes in a unique point. As this point is clearly different from any of the three vertices and from the point L as well, it specifies in each of the three planes $B_i \mathcal{L}^B$ (henceforth simply l-planes) a unique line, and in the $B_1B_2B_3$ -plane (c-plane) a unique, in general proper, conic (see Fig. 9; the three lines and the conic in question being drawn bold). Each of the three pencils of lines thus generates the here-and-there mode, and the pencil of conics features the past-presentfuture pattern. So, our *generic* pencil-borne space-time is, as expected, the space-time as perceived in our *ordinary* state of consciousness.

In order to facilitate our subsequent discussion, we shall compactify our notation for different kinds of pencil-borne patterns. For each elementary pattern we shall reserve one letter, uppercase for time and lowercase for

	CONIC	LINE	PO	INT	_
∞ ⁴	n n				
∞ ³	h h h h				
∞ ²	h				
∞^1		S JASE JASE	μh E		
3			h ho	S.E	
1					μħ μħ
	ALL	3	2	1	NONE

Figure 10. A diagrammatic sketch of an algebraic geometrical classification of pencilborne space-times. Each subfigure features the reference line, the four "fundamental" planes and, in each of the latter, those fundamental elements that are "picked up" by the reference line; the cases where the reference line is incident with all the fundamental lines of a given l-plane and/or with an infinite number of the fundamental conics in the c-plane, are illustrated by drawing several lines and/or conics, respectively. The remaining symbols and notation are explained in the text.

space; a composite mode will then bear several letters, corresponding to the elementary modes it consists of. As for time pencil-patterns, we shall adopt the following symbols: "A" for the ordinary, past-present-future mode (the "arrow" of time); "E" for the present-only ("eternity") mode; and "S" for the no-present (time "standing still") mode. The composite modes will then have the following abbreviations: " μ A·S" for the zero-point mode; " μ A·S·E" for the one-point mode; and "S·E" for the two-point mode, with μ standing for "multi-" and signifying that the number of modes denoted by the letter immediately following this symbol is unlimited/infinite. Concerning space patterns, we shall use "h" for the ordinary, here-and-there mode and "o" for the here-only ("omnipresence") mode. Its composites will accordingly be denoted as " μ h" (vertex-avoiding) and "h·o" (vertex-hitting).

So what are possible kinds of our pencil-borne, "Cremonian" psychospace-times? From an algebraic geometrical point of view, there are altogether 19 different types of them, as depicted in Fig. 10. And they are seen to form a truly remarkable sequence, once being grouped into distinict rows according to their number/abundance (the first column) and into distinct columns according to the number of dimensions of localizability (the lowermost row) and/or the character of the Cremonian image of the reference line in the second projective space (the uppermost row). As easily discernible from the figure, the individual sub-figures differ from each other in the position of the reference line and each of them is accompanied by four of the above-introduced labels/acronyms so that the reader can readily find out the corresponding internal pattern of each spatial dimension and time as well. The number/abundance of a particular type within the structure is, as above, of a geometrical origin. Thus [30], there are ∞^4 (quadruple infinity) of lines in a three dimensional projective space and out of them ∞^3 are incident with a given line, ∞^2 with two different (possibly incident) lines, and ∞^1 pass through a given point and simultaneously lie in a given plane; a line is uniquely defined by two distinct points (their joint) or two different planes (their meet). Non-localizability in a particular dimension means that the reference line does not define a unique line in the corresponding l-plane, or a unique conic in the c-plane; this, obviously, happens when the reference line passes through a base point (or the point L), or lies completely in an l-plane/the c-plane. Hence, "o", " μ h" and "h·o" are nonlocal patterns of space dimensions, while "E", "\(\mu A \cdot S\)," "\(\mu A \cdot S \cdot E\)" and "S\cdot E" are those of time; in Fig. 10, the former/latter are illustrated by drawing several lines/conics in the corresponding l-planes/c-plane so that they can readily be recognized.

A number of intriguing facts can be revealed from Fig. 10. First, and perhaps the most crucial fact, is that our consensus space-time (represented by the sub-figure in the top left-hand corner; this sub-figure is a fully equivalent version of Fig. 9) is, as expected, by far the most abundant type in the hierarchy, as there are ∞^4 of its potential cases. On the other hand, there is just one potential case of space-time where the subject is completely non-localized (the sub-figure in the bottom right-hand corner; the reference line is here identical with the line \mathcal{L}^{B}). Next, it is fairly obvious that the most numerous patterns are those where the subject is completely localized (the "all" column); as the number of dimensions of non-localizability increases (i.e., as we move in the figure from left to right), the number of potential cases decreases (i.e., we move from the top to the bottom of the

figure). Further, we notice that if there are at least two dimensions of non-localizability (the last three columns), one of them is always time. Also, for the two non-ordinary elementary patterns of time, the S-mode prevails over the E-mode. The most variegated row is seen to be the ∞^2 -one (featuring six different types of space-time patterns and spanning three different levels of non-localizability), the least variegated being the top and bottom ones. It is also worth stressing that out of the spatial modes it is the h-one that occurs most frequently, while amongst the temporal patterns it is the S-and E-modes that enjoy this property. Also, there exists no pencil-borne space-time whose space dimensions would be all of the o- or h-o-type. Interestingly, the least frequently encountered patterns are h-o (space) and S-E (time). Finally, there are pairs of patterns which are incompatible with each other: the o-mode with the μ h-one, the S-E-mode with the μ h-one and the S-pattern with the o-one.

From the information gathered in Fig. 10 and the findings of the previous section(s) it will represent no difficulty for the reader to infer and analyze the "experiential contents" for each type of space-time. We only add the following note. One of the most distinguished features of a great majority of extraordinary states of consciousness is a seriously altered sense of individuality, ego, or self-hood. In particular, the greater the departure from our consensus reality, the lesser the sense of ego; ultimately, in the most abstract states, the subject feels to completely transcend/surpass his/her sense of ego, and, so, the dichotomy between subject and object. That important feature, too, has a proper place in our model, once we identify the "degree" of the sense of ego with the level of localization of the subject in our pencil-borne space-times. From Fig. 10 we then readily discern that our consensus experience of space-time is characterized by the strongest sense of the self. As we move across the figure from left to right, the sense of ego accompanying the individual types of space-times (or, better, the corresponding states of consciousness) gradually "melts/dissolves," until it completely vanishes in the state represented by the sub-figure located at the very bottom of the figure. Here is a recently found first-person account that describes in great detail not only this transformation of the sense of "I," but also accompanying profound changes in the perception of both time and space, and which dovetails very nicely with the implications of our model [31]:

For twelve hours I moved in and out of dimensions of both space and time. The incomprehensible became comprehensible. Realities within realities blossomed and faded. From the infinitely large to the infinitely small, unbounded and unfettered mind flashed across landscapes of incredible depth and beauty.... I was looking into the source of my very being, and without question, my creator. And then I came to realize too that I was at the interface between individual mind and absolute mind.

Entheogens, or in my case psilocybes, provide the pivotal role of interfacing between individual consciousness and universal consciousness. It is the crucial link or conduit that bridge the two at a single point. That point then begins to widen, and both entities slowly merge. As the interface grows, what were initially two now opens into one. It's not just a random happening though, an alignment process between the two takes place. Actually it's more a matter of one aligning itself to the other. This is not a conscious operation, although consciousness is witness to it.... To experience this phase of the psychic event was an absolute revelation with all the glory and beauty imaginable. With my minds eye I was able to see the outline of the interface where the two became the One, where duality merged into unity.... I had the pleasant ability at the center of the interface to merge in and out at will. In one moment I was myself, a separate thinking entity with all my individual thoughts; as I merged out my self-hood ceased to exist; my individuality gone; my thoughts as unique things ceased to be, given way to absolute thought. Time and space played an interesting part in this experience. While in myself time existed, time flowed, there was past and future, but while merged in unity time ceased, there was no past or future. Everything was in a single instant; what Plotinus called the "Eternal Now." In myself space had dimension, there was up and down, limitations existed. Merged in the other, there was no up, no down, no limitation, all was infinite and absolute. This gave rise to another incredible phenomenon; with time suspended and space without boundary omniscience came into full awareness; yes, all things known; no limitations to knowledge.... Omnipotence, and omnipresence also became an awesome recognition, but not related to me personally since the I had ceased to be; they were aspects of that great Oneness that was the universe of consciousness. Merging back into my own ego left me with only a memory of being present to it all....

5 Conclusion

A few weeks before his death, in a letter of condolence to the family of his life-time friend Michele Besso, Albert Einstein wrote [32]: "For us believing physicists the distinction between past, present, and future is only an illusion, even if a stubborn one." We have, however, rigorously demonstrated that this "illusion" and its most pronounced "peculiarities/anomalies" are underlaid by a definite algebraic geometrical pattern. Does it mean that our math is a sort of illusion, too? Or, rather, is it physics that falls short of grasping the true nature of time [33]? To tackle this dilemma, we perhaps need a new conceptual framework for physics [34,35], like, for example, the relational blockworld proposed recently by Stuckey et al. [36]. The latter inverts the conventional physics paradigm/hierarchy by taking relations (modelled by matrix variables) to be fundamental to relata (modelled by single-valued variables) and viewing matrix variables as having no counterpart in diachronic/trans-temporal objects, i.e. not having ontological status as "real things." This inverted paradigm is perfectly in the spirit of, and lends support to, our model once we regard individual states of consciousness ("mental space-times") as relations not needing to have counterparts in the "material world," and so in the brain in particular. And this is a truly powerful paradigm shift, supported by a number of extraordinary human experiences, like the following one [37, pp. 26,27]:

I experience myself as beyond everything, literally everything, ...I am a silent witness, vast and unchanging, beyond time and all space...The recognition, which is a direct perception, is that everything is in me. The body, the universe, essence, personality, everything that can become an object of perception, is not me, but is in me. I am pure awareness, mere witnessing, where everything arises and passes away...I am beyond space and time; both space and time are within me. All of time is a movement within me. Personality, or more accurately, the personal experience or soul, is time. Time is the flux of this personal consciousness. Essence is timelessness. I see time as the movement of the timeless within me. All of time, the time of the body and of all of physical existence, is a small process within me. I am beyond time and timelessness. I am the beyond, beyond all and everything.

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